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# ADVANCED SUBSIDIARY GCE

## MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**
None

**Thursday 15 January 2009**
**Morning**

**Duration:** 1 hour 30 minutes

*4755*

**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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**Turn over**

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**Section A** (36 marks)

**1** **(i)** Find the roots of the quadratic equation $z^2 - 6z + 10 = 0$ in the form $a + b\text{j}$. **[2]**

**(ii)** Express these roots in modulus-argument form. **[3]**

**2** Find the values of $A$, $B$ and $C$ in the identity $2x^2 - 13x + 25 \equiv A(x-3)^2 - B(x-2) + C$. **[4]**

**3** Fig. 3 shows the unit square, OABC, and its image, OA′B′C′, after undergoing a transformation.

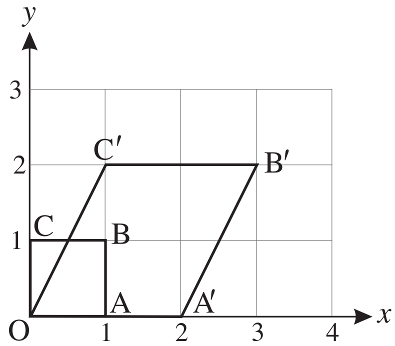


**Fig. 3**

**(i)** Write down the matrix **P** representing this transformation. **[1]**

**(ii)** The parallelogram OA′B′C′ is transformed by the matrix $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$. Find the coordinates of the vertices of its image, OA″B″C″, following this transformation. **[2]**

**(iii)** Describe fully the transformation represented by **QP**. **[2]**

**4** Write down the equation of the locus represented in the Argand diagram shown in Fig. 4. **[3]**

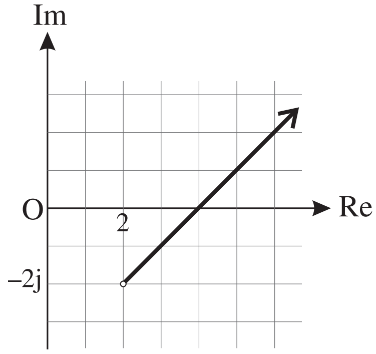


**Fig. 4**

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**5** The cubic equation $x^3 - 5x^2 + px + q = 0$ has roots $\alpha$, $-3\alpha$ and $\alpha + 3$. Find the values of $\alpha$, $p$ and $q$. **[6]**

**6** Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ show that

$$\sum_{r=1}^{n} r(r^2 - 3) = \tfrac{1}{4}n(n+1)(n+3)(n-2).$$ **[6]**

**7** Prove by induction that $12 + 36 + 108 + \ldots + 4 \times 3^n = 6(3^n - 1)$ for all positive integers $n$. **[7]**

**Section B** (36 marks)

**8** Fig. 8 shows part of the graph of $y = \dfrac{x^2 - 3}{(x-4)(x+2)}$. Two sections of the graph have been omitted.

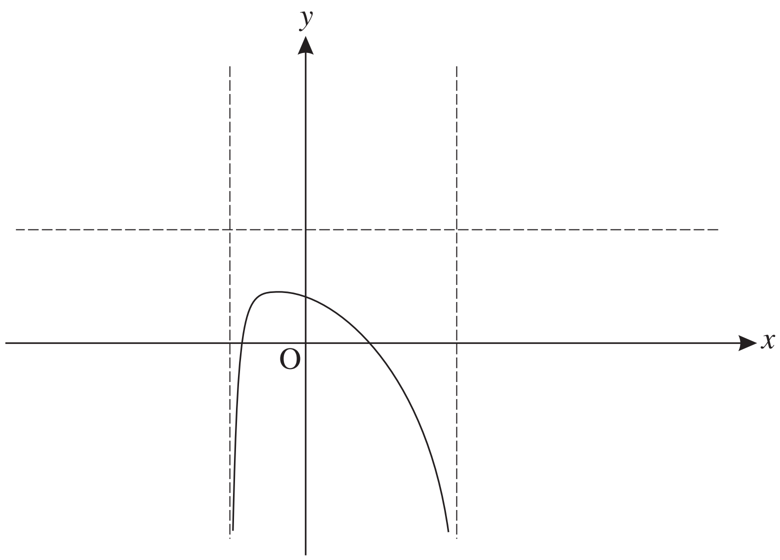


**Fig. 8**

**(i)** Write down the coordinates of the points where the curve crosses the axes. **[2]**

**(ii)** Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

**(iii)** Copy Fig. 8 and draw in the two missing sections. **[4]**

**(iv)** Solve the inequality $\dfrac{x^2 - 3}{(x-4)(x+2)} \leqslant 0$. **[3]**

**[Questions 9 and 10 are printed overleaf.]**

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**9** Two complex numbers, $\alpha$ and $\beta$, are given by $\alpha = 1 + \mathrm{j}$ and $\beta = 2 - \mathrm{j}$.

**(i)** Express $\alpha + \beta$, $\alpha\alpha^*$ and $\dfrac{\alpha + \beta}{\alpha}$ in the form $a + b\mathrm{j}$. **[5]**

**(ii)** Find a quadratic equation with roots $\alpha$ and $\alpha^*$. **[2]**

**(iii)** $\alpha$ and $\beta$ are roots of a quartic equation with real coefficients. Write down the two other roots and find this quartic equation in the form $z^4 + Az^3 + Bz^2 + Cz + D = 0$. **[5]**

**10** You are given that $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 1 & -1 & k \\ -2 & 7 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 5+k \\ -5 & 29 & 7 \end{pmatrix}$ and that $\mathbf{AB}$ is of the form $\mathbf{AB} = \begin{pmatrix} 42 & \alpha & 4k-8 \\ 10-5k & -16+29k & -12+6k \\ 0 & 0 & \beta \end{pmatrix}$.

**(i)** Show that $\alpha = 0$ and $\beta = 28 + 7k$. **[3]**

**(ii)** Find $\mathbf{AB}$ when $k = 2$. **[2]**

**(iii)** For the case when $k = 2$ write down the matrix $\mathbf{A}^{-1}$. **[3]**

**(iv)** Use the result from part **(iii)** to solve the following simultaneous equations. **[4]**

$$\begin{aligned} 3x + 4y - z &= 1 \\ x - y + 2z &= -9 \\ -2x + 7y - 3z &= 26 \end{aligned}$$

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